Introduction to the Math SL Internal Assessment

QUICK FACTS

- Purpose is to enable students to demonstrate the application of their skills and knowledge, and to pursue their personal interests, without the time limitations and other constraints that are associated with written examinations.
- It is an individual exploration that involves investigating an area of mathematics
- The Math IA is a compulsory component of the Math SL course, worth 20% of the final IB grade. If you don't do one, you don't get any grade for Math from the IB (or a Diploma)
- It is a mathematical exploration that should be 6-12 pages in length
- Students should spend approximately 10 hours at home and up to 10 hours in school on this task
- The IA will be graded using 5 criteria (worth a total of 20 points)
- Teachers are only allowed to give advice on one draft. The version submitted after that must be considered the final IA.
- Key Dates
  - ~~Friday 18th Oct~~
    Introduce the requirements of the exploration
    Go through the Criteria
    Talk about choosing a topic
    Hand out Exploration Proposal Form (due Mon 4th Nov which is just after ISSEA)
    Hand out 2 IA's for students to read and grade (over the break)
  - ~~Tuesday 29th Oct~~
    Go through student's grading and give examiners grading
    Talk more about possible topics
  - ~~Monday 4th Nov~~
    IA Topic Proposal DUE
  - ~~Friday 22nd Nov~~
    Full IA Draft DUE (Students must give something in here… but they can do 'hostage exchange' up to the end of the semester 13th December)
  - ~~Friday 20th Dec~~
    I will return your Draft with feedback by this date
  - ~~January Wed 22nd Jan 2014~~
    Final IA DUE

The Exploration

As part of your Mathematics SL course, you will need to write an exploration, which will be assessed and counts as 20% of your final grade. This handout gives you advice on planning your exploration, hints and tips to help you get a good grade by making sure your exploration satisfies the assessment criteria, as well as suggestions on choosing a topic and getting started on your exploration.

1. About the exploration

The Mathematical Exploration is aptly names because your primary objective in writing this report is to explore a topic in which you are genuinely interested and that is at an appropriate level for the course. If you do not submit an exploration then you will receive a grade of 'N" for Mathematics SL, which means you will not receive your IB diploma.

You should aim to spend:

In class time	10 hours of your own time

• Discussing the assessment criteria
• Discussing suitable topics/titles
• Discussing your progress with your teacher
	• Planning your exploration, doing research to help select an appropriate topic
	• Researching, collecting and organising your data and/or information
	• Applying mathematical processes:
	  ▪ Ensuring that all of your results are derived using logical deductive reasoning
	  ▪ Ensuring that your proofs (when necessary) are coherent and correct
	• Demonstrating mathematical communication and presentation:
	  ▪ Checking that your notation and terminology are consistently correct
	  ▪ Adding diagrams, graphs or charts where necessary
	  ▪ Making sure your exploration is clearly structured and reads well

2. Internal assessment criteria

Your exploration will be assessed by your teacher, against given criteria. It will then be externally moderated by the IB using the same assessment criteria. The final mark is the sum of the scores for each criterion. The maximum possible final mark is 20. This is 20% of your final grade for Mathematics SL. A good exploration should be clear and easily understood by one of your peers and self-explanatory all the way through. The criteria are split into five areas, A to E.

Criteria	Title	Points
Criterion A	Communication	4
Criterion B	Mathematical Presentation	3
Criterion C	Personal Engagement	4
Criterion D	Reflection	3
Criterion E	Use of Mathematics	6

Criterion A: Communication

This criterion assesses the organization and coherence of the exploration. A well-organized exploration includes an introduction, has a rationale (which includes explaining why this topic was chosen), describes the aim of the exploration and has a conclusion. A coherent exploration is logically developed and easy to follow.

Graphs, tables and diagrams should accompany the work in the appropriate place and not be attached as appendices to the document.

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	The exploration has some coherence.
2	The exploration has some coherence and shows some organization.
3	The exploration is coherent and well organized.
4	The exploration is coherent, well organized, concise and complete.

Checklist for Criterion A	Yes/No
I believe I expressed my ideas clearly so that my grandmother could understand my work (coherent)	
My ideas are structured in a logical manner so that my classmates can follow the logic (coherent)	
My exploration includes: Introduction, rational, body, conclusion, and bibliography (organized)	
I included all tables and diagrams at appropriate places and not in appendices (organized)	
I cited all references appropriately (organized)	
Every single sentence I wrote is needed and all work I did cannot be ommitted. No repetition! Ms van won't get bored when reading my work; In fact, all my work focused on the aim and notheing irrelevant. (Concise)	
I identified a clear aim for my exploration so my grandmother did not need any clarifications – No missing explanations. (Complete)	

Notes:

Criterion B: Mathematical presentation

This criterion assesses to what extent the student is able to:

- use appropriate mathematical language (notation, symbols, terminology)
- define key terms, where required
- use multiple forms of mathematical representation, such as formulae, diagrams, tables, charts, graphs and models, where appropriate.

Students are expected to use mathematical language when communicating mathematical ideas, reasoning and findings.

Students are encouraged to choose and use appropriate ICT tools such as graphic display calculators, screenshots, graphing, spreadsheets, databases, drawing and word-processing software, as appropriate, to enhance mathematical communication.

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	There is some appropriate mathematical presentation.
2	The mathematical presentation is mostly appropriate.
3	The mathematical presentation is appropriate throughout.

Checklist for Criterion B	Yes/No
I used Microsof 'equation' (or equivalent) to write formulae	
I used the appropriate method for representation (A table when needed, a graph when I am supposed to, spreadsheet when possible. Etc.)	
I revieewed the work and made sure that all key terms are defined	
My resulsts are expressed appropriately. When the result was exact, I used (=) and when it was approximated, I used ≅ or indicated significant figures.	

Notes:

Criterion C: Personal engagement

This criterion assesses the extent to which the student engages with the exploration and makes it their own. Personal engagement may be recognized in different attributes and skills. These include thinking independently and/or creatively, addressing personal interest and presenting mathematical ideas in their own way.

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	There is evidence of limited or superficial personal engagement.
2	There is evidence of some personal engagement.
3	There is evidence of significant personal engagement.
4	There is abundant evidence of outstanding personal engagement.

Checklist for Criterion C	Yes/No
I explained why I took the initiative for working on MY exploration	
MY exploration is related to MY interests and this is explained in MY exploration	
I enjoyed working on MY exploration	
I worked on MY exploration **independently**	
I read about the mathematics used in MY exploration	
I expressed then, the mathematical ideas in MY own way	
I considered historical and global perspectives related to MY exploration	

Notes:

Criterion D: Reflection

This criterion assesses how the student reviews, analyses and evaluates the exploration. Although reflection may be seen in the conclusion to the exploration, it may also be found throughout the exploration.

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	There is evidence of limited or superficial reflection.
2	There is evidence of meaningful reflection.
3	There is substantial evidence of critical reflection.

Checklist for Criterion D	Yes/No
I discussed the implications of my results	
I reflected on the method(s) I used and made links to different fields and/or areas of Mathematics	
I stated possible limitations	
I considered the significance of my exploration	
I discussed possible extensions	
My reflection in the conclusion and within the work went beyond correct interpretations and reached critical interpretations	

Notes:

Criterion E: Use of mathematics

This criterion assesses to what extent students use mathematics in the exploration.

Students are expected to produce work that is commensurate with the level of the course. The mathematics explored should either be part of the syllabus, or at a similar level or beyond. It should not be completely based on mathematics listed in the prior learning. If the level of mathematics is not commensurate with the level of the course, a maximum of two marks can be awarded for this criterion.

The mathematics can be regarded as correct even if there are occasional minor errors as long as they do not detract from the flow of the mathematics or lead to an unreasonable outcome.

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	Some relevant mathematics is used.
2	Some relevant mathematics is used. Limited understanding is demonstrated.
3	Relevant mathematics commensurate with the level of the course is used. Limited understanding is demonstrated.
4	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is partially correct. Some knowledge and understanding are demonstrated.
5	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is mostly correct. Good knowledge and understanding are demonstrated.
6	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is correct. Thorough knowledge and understanding are demonstrated.

Checklist for Criterion E	Yes/No
The mathematics I used is either within the SL syllabus or on a similar level	
I applied mathematics in different contexts	
I reviewed my calculations and they are all correct	
I applied different problem-solving techniques	
I generalized and justified my conclusions	

Notes:

3. Academic Honesty

This is extremely important in all your work. Make sure that you have read and are familiar with the IB Academic Honesty document.

Academic Honesty (in this context) means:

- That your work is authentic
- That your work is your own intellectual property
- That any work taken from another source is properly cited

Authentic work:

- Is work based on your own original ideas
- Can draw on the work and ideas of others, but this must be fully acknowledged (in Citations and Bibliography)
- You must use your own language and expression

4. Record keeping

- Make notes of any books or websites you use, as you go along so you can include them in your bibliography.
- There are different ways of referencing books, websites, etc. Pick a style and use it consistently.
- Keep a record of your actions so that you can show me how much time you are spending on your exploration. Include any meetings you have with me about your exploration.

5. Choosing a topic

You need to choose a topic that interests you, because then you will enjoy working on your exploration, you will put more effort into the exploration and you will be able to demonstrate authentic personal engagement more effectively. You should discuss the topic with your teacher before you put too much time and effort into writing your exploration.

These questions might help you to find a topic for your exploration:

- What areas of the syllabus am I enjoying the most?
- What areas of the syllabus am I performing best in?
- Which mathematical skills are my strengths?
- Do I prefer pure mathematics, or applied problems and modeling?
- Have I discovered, either through reading or the media, mathematical areas outside of the syllabus that I find interesting?
- What career do I eventually want to enter, and what mathematics is important in this field?
- What are my own special interests or hobbies? Where is the mathematics in these areas?

6. Getting started

Over the holidays, you need to narrow down your initial thoughts for your exploration. The next step is to do some initial research. The purpose of the research is to determine the suitability of your topic.

These questions will help you decide if your chosen topic is suitable.

- What areas of mathematics are contained in my topic?
- Which of these areas are accessible to me or are part of the syllabus?
- Is there mathematics outside the syllabus that I would have to learn in order to complete the exploration successfully? Am I capable of doing this?
- Can I show personal engagement in my topic, and how?
- Can I limit my work to the recommended length of 6 to 12 pages if I choose this topic?

Once you think you have a workable topic, you then need to complete one (or 2) proposal forms. (Due Mon 4th Nov). Proposal forms are available on my site (and hard copy).

List of 200 ideas/topics for a Mathematical Exploration

The topics listed here range from fairly broad to quite narrow in scope. It is possible that some of these 200 could be the title or focus of a **Mathematical Exploration**, while others will require you to investigate further to identify a narrower focus to explore. Do not restrict yourself only to the topics listed below. This list is only the 'tip of the iceberg' with regard to potential topics for your Mathematical Exploration. Reading through this list may stimulate you to think of some other topic in which you would be interested in exploring. Many of the items listed below may be unfamiliar to you. A quick search on the internet should give you a better idea what each is about and help you determine if you're interested enough to investigate further – and see if it might be a suitable topic for your Mathematical Exploration.

Algebra and number theory		
Modular arithmetic	Goldbach's conjecture	Probabilistic number theory
Applications of complex numbers	Diophantine equations	Continued fractions
General solution of a cubic equation	Applications of logarithms	Polar equations
Patterns in Pascal's triangle	Finding prime numbers	Random numbers
Pythagorean triples	Mersenne primes	Magic squares and cubes
Loci and complex numbers	Matrices and Cramer's rule	Divisibility tests
Egyptian fractions	Complex numbers and transformations	Euler's identity: $e^{i\pi} + 1 = 0$
Chinese remainder theorem	Fermat's last theorem	Natural logarithms of complex numbers
Twin primes problem	Hypercomplex numbers	Diophantine application: Cole numbers
Odd perfect numbers	Euclidean algorithm for GCF	Palindrome numbers
Factorable sets of integers of the form $ak + b$	Algebraic congruences	Inequalities related to Fibonacci numbers
Combinatorics – art of counting	Boolean algebra	Graphical representation of roots of complex numbers
Roots of unity	Fermat's little theorem	Prime number sieves
Recurrence expressions for phi (golden ratio)		

Geometry		
Non-Euclidean geometries	Cavalieri's principle	Packing 2D and 3D shapes
Ptolemy's theorem	Hexaflexagons	Heron's formula
Geodesic domes	Proofs of Pythagorean theorem	Minimal surfaces and soap bubbles
Tesseract – a 4D cube	Map projections	Tiling the plane – tessellations
Penrose tiles	Morley's theorem	Cycloid curve
Symmetries of spider webs	Fractal tilings	Euler line of a triangle
Fermat point for polygons and polyhedra	Pick's theorem and lattices	Properties of a regular pentagon
Tangrams	Conic sections	Nine-point circle
Geometry of the catenary curve	Regular polyhedra	Euler's formula for polyhedra

Geometry (continued)		
Eratosthenes – measuring earth's circumference	Stacking cannon balls	Ceva's theorem for triangles
Constructing a cone from a circle	Conic sections as loci of points	Consecutive integral triangles
Area of an ellipse	Mandelbrot set and fractal shapes	Curves of constant width
Sierpinksi triangle	Squaring the circle	Polyominoes
Reuleaux triangle	Architecture and trigonometry	Spherical geometry
Gyroid – a minimal surface	Geometric structure of the universe	Rigid and non-rigid geometric structures
Calculus/analysis and functions		
Mean value theorem	Torricelli's trumpet (Gabriel's horn)	Integrating to infinity
Applications of power series	Newton's law of cooling	Fundamental theorem of calculus
Brachistochrone (minimum time) problem	Differential equations	L'Hôpital's rule and evaluating limits
Hyperbolic functions	The harmonic series	Torus – solid of revolution
Projectile motion	Why ***e*** is base of natural logarithm function	
Statistics and modelling		
Traffic flow	Logistic function and constrained growth	Modelling growth of tumours
Modelling epidemics/spread of a virus	Modelling the shape of a bird's egg	Correlation coefficients
Central limit theorem	Modelling change in record performances for a sport	Hypothesis testing
Modelling radioactive decay	Least squares regression	Modelling the carrying capacity of the earth
Regression to the mean	Modelling growth of computer power past few decades	
Probability and probability distributions		
The Monty Hall problem	Monte Carlo simulations	Random walks
Insurance and calculating risks	Poisson distribution and queues	Determination of π by probability
Lotteries	Bayes' theorem	Birthday paradox
Normal distribution and natural phenomena	Medical tests and probability	Probability and expectation
Games and game theory		
The prisoner's dilemma	Sudoku	Gambler's fallacy
Poker and other card games	Knight's tour in chess	Billiards and snooker
Zero sum games		
Topology and networks		
Knots	Steiner problem	Chinese postman problem
Travelling salesman problem	Königsberg bridge problem	Handshake problem
Möbius strip	Klein bottle	
Logic and sets		
Codes and ciphers	Set theory and different 'size' infinities	Mathematical induction
Proof by contradiction	Zeno's paradox of Achilles and the tortoise	Four colour map theorem

Numerical analysis		
Linear programming	Fixed-point iteration	Methods of approximating π
Applications of iteration	Newton's method	Estimating size of large crowds
Generating the number *e*	Descartes' rule of signs	Methods for solving differential equations
Physical, biological and social sciences		
Radiocarbon dating	Gravity, orbits and escape velocity	Mathematical methods in economics
Biostatistics	Genetics	Crystallography
Computing centres of mass	Elliptical orbits	Logarithmic scales – decibel, Richter, etc.
Fibonacci sequence and spirals in nature	Predicting an eclipse	Change in a person's BMI over time
Concepts of equilibrium in economics	Mathematics of the 'credit crunch'	Branching patterns of plants
Column buckling – Euler theory		

Miscellaneous		
Paper folding	Designing bridges	Mathematics of rotating gears
Mathematical card tricks	Curry's paradox – 'missing' square	Barcodes
Applications of parabolas	Music – notes, pitches, scales…	Voting systems
Flatland by Edwin Abbott	Terminal velocity	Towers of Hanoi puzzle
Photography	Art of M.C. Escher	Harmonic mean
Sundials	Navigational systems	The abacus
Construction of calendars	Slide rules	Different number systems
Mathematics of juggling	Global positioning system (GPS)	Optical illusions
Origami	Napier's bones	Celtic designs/knotwork
Design of product packaging	Mathematics of weaving	

Stimuli

Sometimes it is difficult to know where to start with a task as open-ended as this. It is sometimes be useful to start from stimuli and mind map possible ways to explore it. There is a sample 'mind map' on the next page for 'water'. (We'll do a sample mind-map for another one of these stimuli in class).

Psychology	IT/ITGS	Chemistry	Physics
Economics	Business	Biology	Geography
pi	Play	Dance	Health
Viruses	Agriculture	Population	Tiling
Communication	The internet	Codes	Architecture
Symmetry	Games	Euler	Diet
Volcanoes	Food	Orbits	Space
Water	Electricity	e	Motion
Musical harmony	Sine	Music	Cell phones
Algorithms	Computers	Archaeology	sport